Turbulent Skin Friction on Axial Compressor Blading

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Abstract

MEASUREMENTS of turbulent boundary layers in strongly-decelerating flow on axial compressor blades operating at low Reynolds numbers have indicated skin friction values differing by as much as 50% from the predictions of conventional friction laws. A new friction law incorporating strong pressure gradient and longitudinal acceleration effects in the wall layer has been found capable of describing the observed behavior.

Nomenclature

a	= stress gradient, $\partial/\partial y$
p	= static pressure
u	= longitudinal velocity
u_τ	= wall friction velocity, $(\tau_w/\rho)^{1/2}$
x	= streamwise distance
y	= distance normal to surface
A	= stress ratio, τ_0/τ_w
B^+, $B^{+'}$	= law of the wall constants
C_f	= skin friction coefficient, $2\tau_w/\rho U^2$
G	= defect profile shape parameter, $(2/C_f)^{1/2}(1\text{-}1/H)$
H	= boundary-layer shape factor, δ^*/θ
K	= von Kármán constant
Re_c	= chord Reynolds number
Re_{δ^*}	= displacement thickness Reynolds number
Re_θ	= momentum thickness Reynolds number
U	= local freestream velocity
α_0	= pressure gradient parameter, $(\nu/\rho u_\tau^3)\mathrm{d}p/\mathrm{d}x$
α_2,α_3	= stress gradient parameters associated with longitudinal acceleration
δ^*	= boundary-layer displacement thickness
θ	= boundary-layer momentum thickness
μ	= dynamic viscosity
ν	= kinematic viscosity
ρ	= density
τ	= total shear stress
τ_w	= wall shear stress, $\mu(\partial u/\partial y)_{y=0}$
τ_0	= virtual wall shear stress
Π	= equilibrium parameter, $(\delta^*/\tau_w)\mathrm{d}p/\mathrm{d}x$

Subscripts

exp	= experimental value
NM	= value from Nash-Macdonald law

Contents

Measurements by Walker[1] of turbulent boundary layers in strongly decelerating flow on the suction surface of an axial compressor blade under essentially two-dimensional flow conditions at chord Reynolds numbers around 10^5 indicated significant wall shear stress deviations from values predicted by the commonly used Ludwieg-Tillmann[2] and Nash-Macdonald[3] skin friction laws. The experimental observations were made by hot-wire anemometer, and the wall shear stress values were obtained from direct evaluation of the velocity gradient at the wall, with the likely accuracy of dimensionless wall friction velocity u_τ/U generally better than 10%.

Values of skin friction coefficient C_f in turbulent boundary-layer regions as computed from the Ludwieg-Tillmann and Nash-Macdonald laws using measured values of integral boundary-layer parameters Re_θ and H usually agreed to within 10% of each other, but both differed markedly from experiment. The differences between measured and predicted C_f values tended to increase in magnitude with increasing values of the pressure gradient parameter $\alpha_0 = (\nu/\rho u_\tau^3)\mathrm{d}p/\mathrm{d}x$, although they were clearly not functions of α_0 alone. For $\alpha_0 = 1$, deviations from predicted C_f values reached 50%.

The turbulent boundary-layer velocity profiles were characterized by an initial period of relaxation following transition under conditions of moderate pressure gradient, often occurring in association with mid-chord laminar separation bubbles. This was followed by a region of fairly stable or gradually increasing shape factor H, with the trajectory of defect parameter G against equilibrium parameter $\Pi = (\delta^*/\tau_w)\,\mathrm{d}p/\mathrm{d}x$ generally falling about 20% below the equilibrium locus

$$G = 6.1(\Pi + 1.81)^{1/2} - 1.7 \tag{1}$$

given by Nash.[4] There was a complete absence of logarithmic wall similarity in the measured velocity profiles, and only the highest Reynolds number cases showed an incipient tendency to follow the wall asymptote. Better agreement was obtained with McDonald's[5] wall layer model which allows for strong pressure gradient and longitudinal acceleration effects.

The development of a new skin friction law capable of predicting the observed behavior of the compressor blade boundary layers has been described in detail by Walker.[6] The new law is derived from the earlier friction laws of Mellor[7,8] and Nash and Macdonald[3] using the McDonald[5] wall-layer model, which is based on the following shear stress distribution:

1) The stress gradient at the wall is equal to the streamwise pressure gradient.

2) A linear stress distribution

$$\tau = ay + \tau_0 \qquad (a = \text{const}) \tag{2}$$

is assumed in the turbulent wall layer, $a \neq \mathrm{d}p/\mathrm{d}x$ in general.

3) The stress gradient is allowed to vary through the viscous sublayer in response to inertia effects associated with longitudinal acceleration.

4) The wall shear stress τ_w is in general unequal to τ_0.

The turbulent shear stress is described by a mixing length model, with the distribution of the von Kármán constant expressed in terms of a dimensionless coordinate analogous to that of Mellor.[7] The resulting expression for velocity gradient $\partial u/\partial y$ is integrated in conjunction with the assumed shear stress distribution to give velocity $u(y)$ in the wall layer.

The skin friction law of Mellor[7] was developed from the assumption of a stress distribution

$$\tau = (\mathrm{d}p/\mathrm{d}x)y + \tau_w \tag{3}$$

and this law is assumed to remain valid for the case of a

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boundary layer subjected to large pressure gradients and streamwise accelerations, provided that the stress gradient 'a' is substituted for the pressure gradient dp/dx and the skin friction coefficient is based on the apparent wall shear stress τ_0 seen by the turbulent wall layer. The Mellor law is recast in the form of the Nash-Macdonald law (assumed to be the particular case of the Mellor law for α_0 small) and the wall layer model of McDonald is used to relate the apparent and actual wall shear stresses τ_0 and τ_w. This gives the new friction law as

$$(2/C_f)^{1/2} = A^{1/2}\{(1/K)ln(Re_\theta) + f_1(G/A^{1/2}) + [(B^+)' \text{-} 4.75]\} \quad (4)$$

where $A = \tau_0/\tau_w$. McDonald's law of the wall constant in the expression for u/u_τ, is B^+, and $B^{+\prime}$ represents its value for the case $A = 1$ and $\partial\tau/\partial y = dp/dx$. Both A and B^+ vary with α_0 and the ratio of stress gradient to pressure gradient, which in McDonald's theory is related to the parameter $(\alpha_2 + \alpha_3)/\alpha_0$. The function f_1, derived from the Nash-Macdonald law

$$(2/C_{f\text{NM}})^{1/2} = (1/K)ln(Re_\theta) + f_1(\text{G}) \quad (5)$$

is specified by

$$f_1(G) = 1.5G_2 + 1724/(G^2 + 200) \text{-} 12.12 \quad (6)$$

Curves of $C_f/C_{f\text{NM}}$, calculated from Eqs. (4) and (5) for representative values of the various parameters involved, have been plotted in Fig. 1 to compare the predictions of the new skin friction law with the compressor blade measurements of Walker[1] expressed as a ratio of the value predicted by the Nash-Macdonald law. Most of the data presented lies in the range of $300 < Re_\theta < 1100$ and $10 < G < 20$, with absolute values of C_f from 0.001 to 0.003. Curves have been drawn for $Re_\theta = 500$, $G = 10,20$, and $(\alpha_2 + \alpha_3)/\alpha_0 = \text{-}0.5,0,0.5$.

The actual values of stress gradient in the turbulent wall region of the compressor blade are unknown, since no direct turbulent shear stress measurements were obtained. Judging from the observations of other workers, $\partial\tau/\partial y$ should generally lie between 50 and 70% of dp/dx in established turbulent flows proceeding into a region of increasing pressure; this corresponds to values of $(\alpha_2 + \alpha_3)/\alpha_0$ in the range -0.3 to -0.5. The theory of McDonald was used to evaluate the stress gradient indirectly from other data; while individual values were of low accuracy, the average of values in nonrelaxing flow regions was of the order suggested above.

Figure 1 indicates that the new friction law correctly predicts the magnitude of departures from the predictions of the Nash-Macdonald law and the way in which these increase with pressure gradient parameter α_0. Most of the points arise from slowly changing boundary layers with dH/dx small, and these lie around the curves for $(\alpha_2 + \alpha_3)/\alpha_0 = \text{-}0.5$, as expected. Another group of points (partly out of range of Fig. 1) exhibits values of $C_f/C_{f\text{NM}} > 1$. These correspond to flows relaxing after transition with moderate negative dH/dx and have $(\alpha_1 + \alpha_3)/\alpha_0 > 0$ in agreement with the above theory. When H is very large and $dH/dx \ll 0$, both the new friction law and conventional laws break down completely.

There are noticeable Reynolds number effects, with a trend towards poorer agreement as compressor speed is reduced. At 250 rpm, there is a significantly greater scatter in the experimental data. Observations at 150 rpm (corresponding to $Re_c \simeq 3 \times 10^4$) diverged widely from the present theory, and these have been omitted from Fig. 1.

The curve for $(\alpha_2 + \alpha_3)/\alpha_0 = 0$, in which case $A = 1$, corresponds to the friction law of Mellor[7] that allows for pressure gradient effects on the shear stress distribution in the wall layer but does not take into account any changes in stress gradient through the viscous sublayer. This correctly predicts the tendency towards relatively lower values of C_f for established turbulent boundary layers as α_0 increases, but the magnitude of the predicted reduction is far too low. In addition, the Mellor law is incapable of predicting the increase in C_f observed in flows with $dH/dx < 0$ and $(\alpha_2 + \alpha_3)/\alpha_0 > 0$.

Although other published data for strong pressure gradient cases agree qualitatively with the present skin friction model, an accurate quantitative comparison is impossible due to the apparent absence of cases in the literature where significant pressure gradient effects could be expected and where the skin friction was measured directly. Wall shear stress is usually determined indirectly by using a Preston tube or wall-wake fit to the velocity distribution outside the viscous sublayer, and this is unlikely to give the correct value of τ_w when pressure gradient and inertia effects are large.

A further difficulty in comparison arises from the extreme rarity of published cases with pressure gradient parameter $\alpha_0 > 0.1$. Writing

$$\alpha_0 = (2/C_f)^{1/2}\Pi/Re_\delta. \quad (7)$$

and noting that the values of C_f,Π for the present experimental data are quite comparable with those of other published studies, the very high values of α_0 (up to1.0) on the compressor blade can be mainly ascribed to low Reynolds number effects. Thus, it is principally in the case of axial turbomachine blade flows, where low Reynolds number and strong pressure gradients occur in combination, that significant skin friction variations predicted by the present theory can be expected.

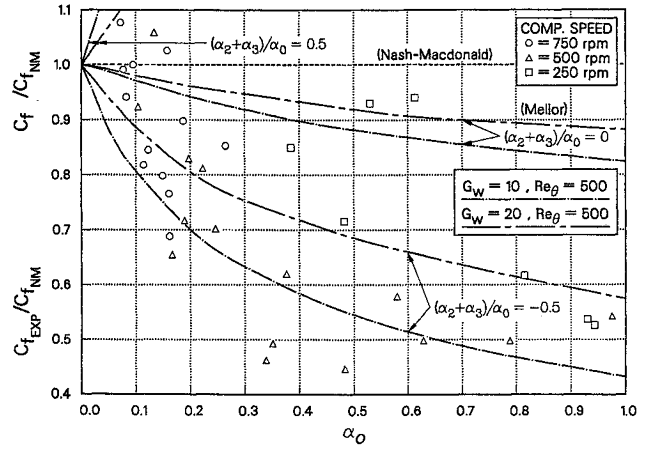

Fig. 1 Comparison of compressor blade skin friction data with predictions of new skin friction law (Nondimensionalized by Nash-Macdonald skin friction values evaluated from measured Re_θ,H).

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